LT1.3: LAPLACE TRANSFORMS

Table of Transforms and Theorems

The fundamental rule for Laplace Transforms is:

$$L[y(t)] = Y(s) = \int_0^\infty e^{-st} y(t)dt$$

Rather than perform what can be a complicated integration, a table is provided with some of the most common transforms already completed:

Transform Table:

$y(t)$	$L[y(t)]=Y(s)$
1	$L[1]=\frac{1}{s}$
Impulse function $\delta(t)$	$L[\delta(t)]=1$
t^n	$L[t^n]=\frac{n!}{s^{n+1}}$
e^{at}	$L[e^{at}]=\frac{1}{s-a}$
$\sin kt$	$L[\sin kt]=\frac{k}{s^2+k^2}$
$\cos kt$	$L[\cos kt]=\frac{s}{s^2+k^2}$
$\sinh kt$	$L[\sinh kt]=\frac{k}{s^2-k^2}$
$\cosh kt$	$L[\cosh kt]=\frac{s}{s^2-k^2}$
$\frac{dy}{dt}$	$L\left[\frac{dy}{dt}\right]=sY(s)-y(0)$
$\frac{d^2y}{dt^2}$	$L\left[\frac{d^2y}{dt^2}\right]=s^2Y(s)-sy(0)-y'(0)$

Laplace Transform Operational Theorems

Below are listed some of the commonly used operational theorems in Laplace Transform. This list is not comprehensive, and there are other theorems not listed here.

$af(t)+bg(t)$	$L[af(t)+bg(t)]=aF(s)+bG(s)$	$a,b\in\Re$
$f(at)$	$L[f(at)]=\frac{1}{a}F(s)$	$a>0$
$e^{at}f(t)$	$L[e^{at}f(t)]=F(s-a)$	
$f(t-\tau)H(t-\tau)$	$L[f(t-\tau)H(t-\tau)]=e^{-\tau s}F(s)$	
$t^n f(t)$	$L[t^n f(t)]=(-1)^n\frac{d^n F(s)}{ds^n}$	
$\frac{f(t)}{t}$	$L\left[\frac{f(t)}{t}\right]=\int_s^\infty F(u)du$	$F(u)=L[f(t)]$
$\frac{d}{ds}f(t)$	$L\left[\frac{d}{ds}f(t)\right]=sF(s)-f(0)$	
$\frac{d^2}{ds^2}f(t)$	$L\left[\frac{d^2}{ds^2}f(t)\right]=s^2F(s)-sf(0)-f'(0)$	